

HOMEWORK 6

Instruction: Please complete the following 8 problems. Please print the homework and work on it directly (It is up to you whether to print it double sided or not). Due on Friday, Feb 24

Problem 1. One quarter of the five-element subsets of $\{1, 2, \dots, n\}$ contains the element 7. Determine n .

Problem 2. How many subsets of $\{1, 2, \dots, 11\}$ contain at least one even integer?

Problem 3. For $A = \{1, 2, 3, 4, 5, 6, 7\}$, determine the number of

- (1) subsets of A
- (2) nonempty subsets of A
- (3) subsets of A containing 1,2
- (4) subsets of A containing five elements, including 1,2
- (5) subsets of A with an even number of elements

Problem 4. Determine the number of positive integer n where $1 \leq n \leq 100$ such that n is not divisible by 2,3,5.

Problem 5. In how many ways can the 26 letters of the alphabet be permuted so that none of the patterns **math**, **cs**, **physics** occurs?

Problem 6. Determine $|A \cup B \cup C|$ when $|A| = 50, |B| = 500, |C| = 5000$ for the following cases:

- (1) $A \subset B \subset C$;
- (2) $A \cap B = A \cap C = B \cap C = \emptyset$
- (3) $|A \cap B| = |A \cap C| = |B \cap C| = 3$
- (4) $|A \cap B \cap C| = 1$.

Problem 7. How many permutations of the digits $0, 1, \dots, 9$ either start with a 3 or end with a 7?

Problem 8. Read Example 1.42, Chapter 1.5 carefully, then mimic the proof to show part (a) in Example 1.43.