

Applied Combinatorics, Section N2

Test 2

Name: _____

GTID: _____

<i>Problem</i>	<i>Points</i>
1	
2	
3	
4	
5	
6	

TOTAL: _____

Please do show all your work including intermediate steps and also explain in words how you solve a problem. Partial credits are available.

Problem 1. What is the coefficient of x^6 in $(x^2 + x)^5$.

Problem 2. A closet contains 10 pairs of shoes. If 8 shoes are randomly selected, how many ways are there such that

- (1) at least one pair are chosen;
- (2) no complete pair are chosen.

Problem 3. Determine the number of positive integer x where $x \leq 999$ and the sum of the digits in x equals 23.

Problem 4. In how many ways can one toss 100 dice so that all 6 types of face will be showing?

Problem 5. Let m be a positive odd integer. Show that there exists a positive integer n such that $m \mid 2^n - 1$ (Hint: first show that there must exist two integers i, j such that $2^i \equiv 2^j \pmod{m}$).

Problem 6. Compute

$$\sum_{k=0}^n \binom{n}{k} k.$$